Assessment of a Closed-Form Iterative Water Filling Energy Efficient Power Control Algorithm in Multi-Carrier Context

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***Abstract*—In this article, we investigate energy efficient power control for the uplink of an OFDMA single-cell communication system. The user equipments decide freely their power level on the available subcarriers of the system in order to maximize their EE (Energy Efficiency) while respecting minimum normalized rate requirements. We use the non-cooperative Game theory to demonstrate that users will end by learning the equilibrium. The solution of the Game is given as a water-filling like power allocation. Furthermore, we implement a distributed power allocation scheme using two ways: *a*) Existing Algorithm: power allocation is done by using Fractional Programing techniques *b*) Proposed Algorithm: power allocation is done by using a closed form expression.**

Simulations show that even though the proposed algorithm has less EE than the reference algorithm, it performs better in terms of SE (Spectral Efficiency).

***Index Terms*—Energy Efficiency, Power Control, non-cooperative Game Theory, Resource Allocation, OFDMA, uplink**

I. INTRODUCTION

Nowadays Telecommunication industry is facing a tremendous growth of data usage driven by energy-hungry applications (e.g: video streaming, on-line gaming, mobile P2P, etc.). Indeed, the mobile data traffic is growing at a 57 percent CAGR (Compound Annual Growth Rate) from 2014 to 2019 [1] which corresponds to a tenfold increase every 5 years. From end user perspective, battery life is a determinant factor of mobile data Quality of Experience [2]. To keep up with mobile data growth, more battery technology improvement is needed. Unfortunately, battery technology is not evolving that fast [3].

Consequently, enhancing the EE (Energy Efficiency) of wireless communication becomes crucial for reducing the gap between the exponential data consumption and the relatively slow battery technology development. Even though a breakthrough happened in battery technology, economical, environmental concerns and social responsibility of end users would continue to be a motivation of higher EE. Despite limited ICT(Information and Communication Technologies) contribution to the global CO_2 emissions around 2%, the ICT related CO_2 emissions are increasing 10% on annual base [4]. Energy consumption of mobile networks is growing much faster than ICT on the whole [2]. As a result, more efforts are required in the Energy Efficient Wireless Communication field in order to limit the environmental footprint of Mobile Telecommunication industry.

Increased intelligence and higher processing capabilities of modern User Equipment (UE) paved the way to new distributed paradigms like Cognitive Radio and Ad-hoc Networks. Moreover, the massive small cell deployment and the continuous shrinking cell size implies less UEs per Base Station (BS). Then, decreasing the processing burden on the BS side is cogent and convenient to increase system scalability and decrease complexity.

Distributed Energy Efficient power control design allows wireless users to enjoy a better quality of experience by increasing their battery lifetime for the same amount of data transmitted while enhancing the access to shared medium and decreasing the complexity handled by the BS.

II. RELATED WORKS & ARTICLE CONTRIBUTIONS

In the seminal work of [5] authors define the EE metric often encountered in subsequent works, data successfully transmitted (bit) over energy consumed (Joule). In [3] authors investigate energy efficient power optimization scheme for interference limited environment using non-cooperative games and then derive the trade-off between EE and spectral efficiency (SE).

The work of [6] proposes a power allocation scheme for maximizing the energy efficiency while respecting minimum QoS of the uplink of an OFDMA based HetNet where a macro cell is aided by a set of small cell. The authors propose a solution concept based on Debreu Equilibrium which is a generalization of Nash Equilibrium, an algorithm based on water-filling best response. In [7] authors investigate the trade-off between energy efficiency (EE) and spectral efficiency (SE) in device-to device (D2D) communications underlaying cellular networks with uplink channel reuse.

In this article, we derive an uplink Energy Efficient communications scheme for a single cell multi subcarriers system with minimum QoS requirement. We used a non-cooperative game formulation to maximize the EE. We compared two ways of dealing with EE of the network: *a*) Existing Iterative power

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allocation Algorithm using Dinkelbach Method b) Proposed Iterative power allocation Algorithm based on closed form . As far as we know the proposed closed form expression exists in the literature but was never assessed and implemented. So the novelty of this paper is the assessment and the implementation of the closed form expression algorithm. Indeed in [8], closed expression of power was expressed theoretically but was not implemented. We represent and compare graphically the EE and system throughput as functions both Closed-Form and Dinkelbach algorithms.
The remainder of this paper is organized as follows. In section IV, we describe the system model. The proposed power control game is discussed in section V and its Generalized Nash equilibrium solution is derived. In section VI algorithms design is given and practical considerations are described. Section VII results obtained from computer simulation are commented. Finally, we give conclusions in section VIII.

III. Notations

In this section we define the notations we adopt in this paper:

- x^+ stands for $max(0, x)$
- Matrices and vectors are written in bold letters
- $\mathcal{W}()$ denotes the Lambert W function
- For a vector: $(\alpha_1, \ldots, \alpha_K)$ we denote it in a short form as (α_k, α_{-k}) with:

$$\alpha_{-k} = (\alpha_1, \ldots, \alpha_{k-1}, \alpha_{k+1}, \ldots, \alpha_K)$$

- The cardinality of a set A is denoted: $|A|$

IV. System Model

The network is composed of one BS (Base Station) and K cellular UEs. The set of UEs in the system is denoted as:

$$\mathcal{K} = \{UE_1, UE_2, \ldots, UE_K\}$$

We have adopted a single cell network based on OFDMA (Orthogonal Frequency Division Multiple Access) with the following spectral resources:

- Bandwidth: W
- Number of sub-carriers: N
- Bandwidth per sub-carrier is: $B = \frac{W}{N}$.

The noise power: $\sigma^2 = B * N_0$, with N_0 the noise spectral density. Every UE has the freedom to transmit on all subcarriers and to choose his power level.
The parameters of user k are as follows:

- $h_{k,n}$: the channel gain between the cellular UE_k and the BS over the n^{th} subcarrier
- $p_{k,n}$: the transmit power of the cellular UE_k over the subcarrier n.
- p_{max} : the maximum power per subcarrier.
- p_{cir} : the circuit power linked to electronics consumption.
- R_{min} : the minimum normalized rate requirement.

For each UE_k we denote its power vector on all subcarriers:

$$\mathbf{p_k} = [p_{k,1}, p_{k,2}, \ldots, p_{k,N}]$$

The total power consumption of UE_k taking into account the radiative power all over the subcarriers and circuit power:

$$P_{tot}(\mathbf{p_k}) = \sum_{n=1}^{N} p_{k,n} + p_{cir}$$

And the vector of powers other than UE_k powers is denoted:

$$\mathbf{p_{-k}} = [\mathbf{p_1}, \ldots, \mathbf{p_{k-1}}, \mathbf{p_{k+1}}, \ldots \mathbf{p_K}]$$

We denote the SINR (Signal-to-Interference-plus-Noise Ratio) of UE_k over the subcarrier n:

$$\gamma_{k,n} = \frac{h_{k,n} p_{k,n}}{\sum_{\substack{i=1 \\ i \neq k}}^{K} h_i p_{i,n} + \sigma^2}$$

Let us define the effective channel gain as following:

$$\nu_{k,n} = \frac{\gamma_{k,n}}{p_{k,n}} = \frac{h_{k,n}}{\sum_{\substack{i=1 \\ i \neq k}}^{K} h_i p_{i,n} + \sigma^2}$$

The utility function corresponding to the SE (Spectral Efficiency) of the uplink communication of UE_k is :

$$U_k^{SE}(\mathbf{p_k}, \mathbf{p_{-k}}) = \sum_{n=1}^{N} log_2(1 + \gamma_{k,n})$$

The utility function corresponding to the EE (Energy Efficiency) of the uplink communication of UE_k is :

$$U_k^{EE}(\mathbf{p_k}, \mathbf{p_{-k}}) = \frac{U_k^{SE}(\mathbf{p_k}, \mathbf{p_{-k}})}{P_{tot}(\mathbf{p_k})}$$

The UE will try to maximize its global energy efficiency while respecting minimum normalized rate constraint. The optimization problem that the UE will try to solve is the following:

$$\begin{aligned} \underset{\mathbf{p_k}}{\text{Maximize}} \quad & U_k^{EE}(\mathbf{p_k}, \mathbf{p_{-k}}) \\ \text{s.t.} \quad & \text{(C1) } \mathbf{p_k} \in [0, p_{max}]^N \\ & \text{(C2) } U_k^{SE}(\mathbf{p_k}, \mathbf{p_{-k}}) \geq R_{min} \end{aligned} \tag{1}$$

Consequently, in our system there are K optimization problems that have to be solved jointly. One might wonder why not to apply directly the multivariate Optimization Theory instead of Game Theory. The essential difference between multi-variable optimization and game theory is that in the former a centralized entity has full control on all variables but in the latter each player has partial control on his own variables.

V. Game Theory Formulation

As each UE chooses freely its power levels over the subcarriers, we have then rational agents sharing the same resources controlling only their variables but their outcome depends on all players' actions. The natural framework to study interactive decision making between rational agents is the non-cooperative game theory. The non-cooperative game is formally defined as the following triplet:

$$\mathcal{G} = \{\mathcal{K}, \{\mathcal{A}_k\}_{k \in \{1,\ldots,K\}}, \{\mathcal{U}_k\}_{k \in \{,1,\ldots,K\}}\}$$

where:

- Players: $\mathcal{K} = \{UE_1, UE_2, \ldots, UE_K\}$
- Actions: $\mathcal{A}_k = [0, p_{max}]^N$, set of UE_k actions
- Utilities: The utility function corresponds to normalized EE expressed in $(bit/J/Hz)$.

For each $k \in \{1, \ldots, K\}$, let us denote the action space of the other players:

$$\mathcal{A}_{-k} = \prod_{\substack{i \in \{1,\ldots,K\} \\ i \neq k}} \mathcal{A}_i$$

Now we have to determine the equilibrium of the game $\mathcal{G}$. The existence and uniqueness of equilibrium is very important for distributed system design because when agents reach equilibrium they have stable and predicable behavior. It is at the equilibrium that network will effectively operate [9]. We use the solution concept of Debreu Equilibrium also named GNE (Generalized Nash Equilibrium). GNE problem is a non-cooperative game in which each player's admissible set of strategies depend on the other players' strategies. The difference between GNE and NE that in latter the player can deviate to whatever strategy that maximizes the utility, but in the former the strategy must respect the constraint. In other words, the power profile of the player must be in the feasible action set that respect the minimum normalized rate constraint and maximum power constraint. We will denote by $\mathcal{A}_k^G(\mathbf{p_{-k}})$ the set of power of user UE_k that respect of the minimum normalized rate constraint and maximum power constraint knowing other players strategy .

$$\mathcal{A}_k^G(\mathbf{p_{-k}}) = \{\mathbf{p_k} \in \mathcal{A}_k : \mathbf{p_k} \text{ verifies (C1) \& (C2)}\}$$

,

Definition. *Generalized Nash Equilibrium (Debreu Equilibrium)*
A joint power strategy matrix $\mathbf{p}^* = (\mathbf{p_k^*}, \mathbf{p_{-k}^*})$ is said to be a Nash equilibrium if:

$$\mathbf{p_{-k}^*} \in \mathcal{A}_k^G(\mathbf{p_{-k}^*})$$

and:

$$\forall \mathbf{p_k} \in \mathcal{A}_k^G(\mathbf{p_{-k}^*}) : U_k^{EE}(\mathbf{p}^*) \geq U_k^{EE}(\mathbf{p_k}, \mathbf{p_{-k}^*})$$

In other words , when the players are playing a GNE no player has interest to deviate unilaterally from this equilibrium to another feasible point.

The objective function defined in (1) is non-concave, we use fractional programming to transform it into quasi-concave problems. This same method was used for example in [7].

Proposition 1. *(Necessary Conditions) When the Game admits a GNE the optimal power of the UEs is given by the following formula:*

$$\forall k \in \mathcal{K} : p_{k,n}^* = min\left(p_{max}, \left(\frac{1}{\lambda_k^*} - \frac{1}{\nu_{k,n}}\right)^+\right) \qquad (2)$$

Algorithm 1: Iterative Power control

1) **Tolerance** $\Delta \ll 1$
2) **Input** : $t = 1$, $\mathbf{p_k}(t) = \mathbf{p_{init}}$ for $k \in \{0, \ldots, K\}$
3) **Loop over UE** for $k \in \{1, \ldots, K\}$
   a) **Receive** $\gamma_{\mathbf{k}}(t-1)$
   b) **Calculate** $\nu_{\mathbf{k}}(t)$
   c) **While** $||\mathbf{p_k}(t+1) - \mathbf{p_k}(t)|| > \Delta$
      i) **Calculate** λ_k^{max} using **Algorithm 2**
      ii) **Calculate** λ_k using **Algorithm 3** or **Algorithm 4**
      iii) **Calculate** λ_k^* as in (3)
      iv) **Calculate** $\mathbf{p_k}(t+1)$ as in (2)
4) **Return** $\mathbf{p_k}(t)$ for $k \in \{1, \ldots, N\}$

The optimal value λ_k^*:

$$\lambda_k^* = min(\lambda_k, \lambda_k^{max}) \qquad (3)$$

$$\lambda_k = \frac{\mathcal{W}(B_k \exp(A_k - 1))}{B_k} \qquad (4)$$

Let us denote N_k^*, the set of active subcarriers of the user UE_k (i.e. with strictly positive power)when $\lambda_k^* = \lambda_k$.

$$N_k^* = \left\{ n \in N \mid \left(\frac{1}{\lambda_k} - \frac{1}{\nu_{k,n}}\right) > 0 \right\}$$

With:

$$A_k = \frac{log\left(\prod_{n \in N_k^*} \nu_{k,n}\right)}{|N_k^*|} \qquad (5)$$

$$B_k = \frac{\left(p_{cir} - \sum_{n \in N_k^*} \frac{1}{\nu_{k,n}}\right)}{|N_k^*|} \qquad (6)$$

λ_k^{max} is obtained when the normalized rate constraints are active:

$$\lambda_k^{max} = \left(2^{-|N_k^{max}|R_{min}} \prod_{n \in N_k^{max}} \nu_{k,n}\right)^{\frac{1}{|N_k^{max}|}} \qquad (7)$$

$$N^{max} = \left\{ n \in N \mid \left(\frac{1}{\lambda_k^{max}} - \frac{1}{\nu_{k,n}}\right) > 0 \right\}$$

Proof. The proof is given in Appendix A, the proposition and the proof are largely inspired from [6] applied to our model. We take into consideration the maximum power constraint in the expression of allocated power. ☐

VI. Algorithms Design

Practically, each UE will solve the optimization problem (1) by giving its best response to the strategy of the other players. The Best Response function of the UE_k is the best action $\mathbf{p_k}$ to adopt knowing other players' actions $\mathbf{p_{-k}}$:

$$\forall \mathbf{p_{-k}} \in \mathcal{A}_{-k}, \mathrm{BR_k}(\mathbf{p_{-k}}) = \mathrm{argmax}_{\mathbf{p_k}} U_k^{EE}(\mathbf{p_k}, \mathbf{p_{-k}})$$

As instantaneous feed back is not possible in practice, then we adopt an asynchronous scheme where the user choose a

Algorithm 2: Inverse Water Filling for user k

1) **Initialize** $N_k^* = N$
2) **Initialize** λ_k^{max} as in (7)
3) **Loop over subcarrier** for $n \in \{1, \ldots, N\}$
   a) **Calculate** $p_{k,n}$ as in (2)
   b) **If** $p_{k,n} = 0$, $N_k^* = N \setminus \{n\}$
   c) **Else** continue
4) **Update** λ_k^{max} as in (7)
5) **Return** λ_k^{max}

power vector at step t based on the actions taken by the other users in the previous step $t-1$. Actually, each user transmitter receives the SINR all over the subcarriers from the Base Station which is the only information available to the transmitter together with his individual CSI (Channel State Information)

In the first step of the algorithm the user chooses the initial vector of power $\mathbf{p_{init}}$. In each step, the user UE_k calculates the estimated effective channel gain using the value of the previous SINR on subcarrier n:

$$\widehat{\nu_{k,n}}(t) = \frac{\gamma_{k,n}(t-1)}{p_{k,n}(t-1)}$$

The estimated utility function that the user UE_k will try to maximize w.r.t to conditions (C1) and (C2):

$$\widehat{U_k^{EE}}(\mathbf{p_k}(t), \mathbf{p_{-k}}(t-1)) = \frac{\sum_{n=1}^{N} log_2(1 + \widehat{\nu}_{k,n}(t) p_{k,n}(t))}{\sum_{n=1}^{N} p_{k,n}(t) + p_{cir}}$$

As in proposition 1:

$$\forall k \in \{1, \ldots, K\} : p_{k,n}^*(t) = \left(\frac{1}{\lambda_k^*(t)} - \frac{1}{\widehat{\nu_{k,n}}(t)} \right)^+ \quad (8)$$

The challenge here is to find $\lambda_k^*(t)$, for that we use two methods:

- The first method that was used by [6] relies on Dinkelbach algorithm: **Algorithm 3**.
- The second method is the proposed in this article relies on the closed form expression as in **Algorithm 4**.
- The third method is the IWF (Inverse Water Filling) is applied using**Algorithm 2** which aims to have exactly the minimum normalized SE (R_{min}).

Algorithm 3: Dinkelback method to find λ_k

1) **Tolerance** $\epsilon \ll 1$
2) **While** $(U_k^{SE} - \lambda_k P_{tot}) < 0$
   a) **Select** Random $\lambda_k \in \mathbb{R}^+$
   b) **Calculate** (Loop over subcarriers) $p_{k,n}$ as in (7)
   c) **Compute** $U_k^{SE}(\mathbf{p_k})$ and $P_{tot}(\mathbf{p_k})$
   d) **Update** $U_k^{SE} - \lambda_k P_{tot}$
3) **End While**
4) **While** $(U_k^{SE} - \lambda_k P_{tot}) > \epsilon$
   a) **set** $\lambda_k = U_k^{SE}/P_{tot}$
   b) **Calculate** (Loop over subcarriers) $p_{k,n}$ as in (7)
   c) **Update** $U_k^{SE}$and P_{tot}
5) **End While**
6) **Return** λ_k

Algorithm 4: Closed-form expression to find λ_k

1) **Initialize** $N_k^{max} = N$
2) **Initialize** λ_k as in (4)
3) **Loop over subcarrier** for $n \in \{1, \ldots, N\}$
   a) **Calculate** $p_{k,n}$ as in (2)
   b) **If** $p_{k,n} = 0$, $N_k^{max} = N \setminus \{n\}$
   c) **Else** continue
4) **Update** λ_k as in (4)
5) **Return** λ_k

VII. Simulation and comments

In order to compare Algorithm 3 (Dinkelbach) and Algorithm 4 (Closed-Form), we use a single cell system where the BS is located at center of the cell that has 300 meters radius and $K = 10$ cellular users have uplink communication towards BS.

The Table (I) present main parameters adopted for the simulation. The propagation exponent mentioned in the table α serves to calculate the channel gain as follow: $h = \frac{cte}{d^\alpha}$ which accounts the path-loss. We choose $\mathbf{p_{init}} = 1|_N \times p_{max}/2$ as initial power vector for all users. We choose this because it is the mean of the minimum power and the maximum power. We choose a reference user UE_1 with a fixed position $[50, 50]$ and the other 9 users are generated randomly over the cell. We set $d_{min} = 20$ meters the minimum distance from the BS. For a given realization of other $K-1$ users, all K users learn the equilibrium by performing iteratively the Algorithm 3 or Algorithm 4 for 30 iterations to be sure of convergence. The results of simulation we obtain are the Energy Efficiency, Spectral Efficiency and Power per sub-carrier of the reference user as function of iterations averaged on the number of the other users positions 1000 realizations.

We deduce from the simulations that equilibrium of the game exists as all algorithms converge before the 10^{th} iteration.

Also the maximum power is not reached so we have a non-saturated equilibrium. This is important, because in case of saturated equilibrium the power at equilibrium is p_{max} and may be if the condition of maximum power is relaxed the Game will not reach equilibrium.

For all simulations we choose a flat fading channel. Consequently, the power per subcarrier is equal for all subcarriers. In term of minimum normalized rate respect, we remark that

TABLE I
LIST OF PARAMETERS

Notation	Meaning	Value
N	Number of subcarriers	5
N_0	Noise spectral density	$3.98 \times 10^{-19} W/Hz$
B	Bandwidth per subcarrier	$1MHz$
p_{cir}	Power circuit	$300mW$
α	Propagation exponent	3.6
cte	Constant of propagation	$2.57399 * 10-2$
d_{min}	Minimum Distance	20 meters
R_{min}	Minimum Normalized Rate	1.5 bit/s/Hz
p_{max}	Maximum power/subcarrier	0.2 W

both Algorithm 3 & 4 respect minimum SE: $1.5bit/s/Hz$ to the reference user. So both algorithm converge to a solution that respects the minimum normalized rate constraint. By construction the IWF(Inverse Water Filling) algorithm respect the minimum normalized rate constraint as the powers are calculated to achieve exactly $1.5bit/s/Hz$.
The outcome of both Algorithm 3 & 4 respect the condition (C1) and (C2) in the maximization problem (1).

When at equilibrium, each user chooses to send at fixed power over all the subcarriers. This is a direct result of the channel gain flat fading assumption: there is no better subcarrier to transmit for the user. We give below our comments on results obtained:

- **EE**: The Energy efficiency corresponds to the EE utility times the bandwidth. The simulations in Figure 2 that represent the EE shows that the Algorithm 3 is 16% better than Algorithm 4
- **SE**: simulations in Figure 1 shows that the SE of the Algorithm 4 is nearly 40% better than Algorithm 3
- **Power**: simulations in Figure 3, shows that the original algorithm is better again in term of power per subcarrier: a user using the Algorithm 3 will consume almost the half of Energy used when choosing Algorithm 4. The IWF algorithm has the minimum power consumption as it aims to achieve only the minimum normalized rate.

VIII. CONCLUSION & FUTURE WORKS

In this work we propose to use a closed-form expression of the power allocation instead of using Dinkelbach algorithm. Results shows that the proposed closed-form algorithm gives better SE (Spectral Efficiency) than the existing Dinkelbach Algorithm, the EE is less than the original Algorithm. However, the power consumption is much higher without going beyond maximum power constraint. Consequently as this Algorithm gives good insights, more advanced assessment techniques need to be in future: complexity, scalability to increase of number of users, etc. Also, more investigation need to be done in order to increase its Energy Efficiency. Another

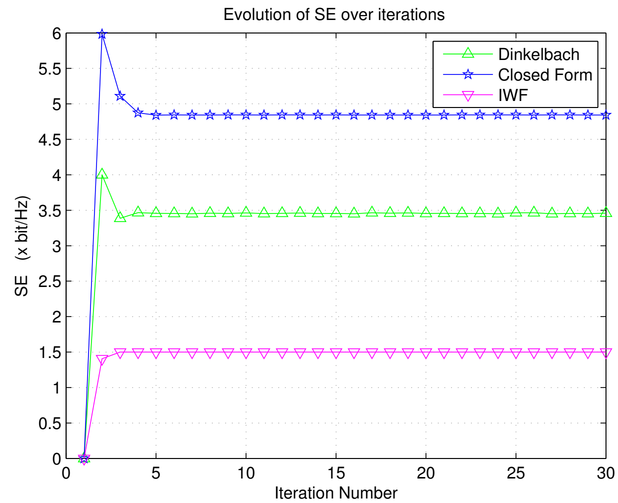

Fig. 1. Comparison in term of Spectral Efficiency

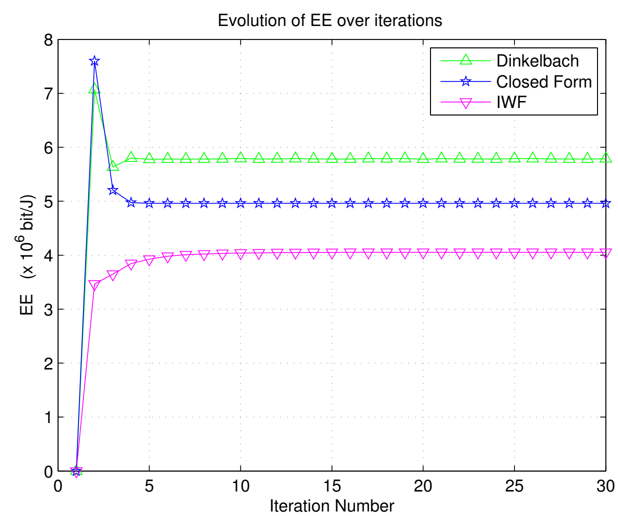

Fig. 2. Comparison in term of Energy Efficiency

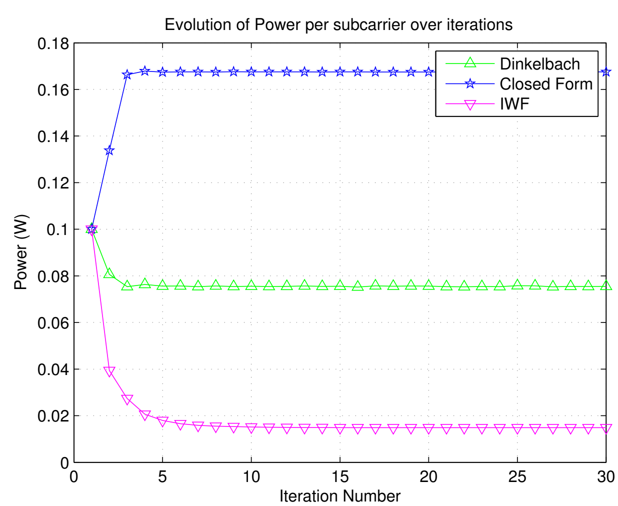

Fig. 3. Comparison in term of Power Consumption

possible future work is the application to more sophisticated scenarios such as Device To Device (D2D) and Small Cells.

APPENDIX A
PROOF OF GNE PROPOSITION

First, we give here a brief introduction to Fractional programing:

Definition. *Fractional programing*
Let $C \subset \mathbb{R}^n$ a convex subset and let $C \mapsto \mathbb{R}$ following functions:

$$f : x \mapsto f(x) \qquad g : x \longmapsto g(x)$$

A fractional program is the optimization problem:

$$\underset{x \in C}{Maximize} \quad \frac{f(x)}{g(x)} \tag{9}$$

Proposition 2. *The vector $x^* \in C$ is a solution to the equation (9) if and only if: $f(x^*) - \lambda^* g(x^*) = 0$, λ^* is the zero of the function:*

$$H(\lambda) = \underset{x \in C}{Maximize} \; f(x) - \lambda g(x)$$

For a given value λ_k, the player k will solve the following transformed problem instead of (1):

$$\begin{aligned} &\underset{\mathbf{p_k}}{\text{Maximize}} && U_k^{SE}(\mathbf{p_k}, \mathbf{p_{-k}}) - \lambda_k P_{tot}(\mathbf{p_k}) \\ &\text{s.t. (C1) \& (C2)} \end{aligned} \tag{10}$$

Now the objective function of the problem is concave as log function and affine function are both concave. We remind that the sum of concave functions is a concave function. As the conditions are also concave, we can use standard convex programming. When the equilibrium is achieved the first order derivative of the transformed objective function using fractional programing is equal to zero (necessary condition). That gives directly the value of $p_{k,n}^*$. We have:

$$\sum_{n=1}^{N} log_2(1 + \nu_{k,n} p_{k,n}^*) = \lambda_k^* (\sum_{n=1}^{N} p_{k,n}^* + p_c ir)$$

This gives us by using the expression of $p_{k,n}^*$:

$$\sum_{n \in N_k^*} log_2(\frac{\nu_{k,n}}{\lambda_k^*}) = \lambda_k^* (\sum_{n \in N_k^*} p_{k,n}^* + p_{cir})$$

After rearranging, we use A_k and B_k as in (5) & (8) :

$$\exp(\lambda B_k) \lambda B_k = \frac{\exp(A_k - log(2))}{B_k}$$

Exploiting the following property of Lambert W function:

$$\exp(\mathcal{W}(x)) \mathcal{W}(x) = x$$

We find the expression of λ_k as in (4). Now let us move to λ_k^{max} expression, when the condition (C1) is an equality the problem becomes minimizing P_{tot} subject to the equality:

$$U_k^{SE} = R_{min}$$

In a straightforward way we obtain the expression (7). Let us prove now that λ_k^{max} is the maximum bound of λ_k Let us denote :

- N_k^{max} the set of the active subcarriers when the condition (C1) is active
- N_k the set of the active subcarriers when the condition (C1) is not active for a given λ_k

$$U_k^{SE}(\mathbf{p_k}) > R_{min}$$

We replace $p_{k,n}$ by its WF value we get:

$$\sum_{n \in N_k} log_2\Big(\frac{\nu_{k,n}}{\lambda_k}\Big) > \sum_{n \in N_k^{max}} log_2\Big(\frac{\nu_{k,n}}{\lambda_k^{max}}\Big)$$

Suppose that :

$$\lambda_k^{max} < \lambda_k$$

$$\forall n \in N_k : \lambda_k^{max} < \lambda_k < \nu_{k,n}$$

So $N_k \subset N_k^{max}$ Then :

$$\sum_{n \in N_k^{max}} log_2\Big(\frac{\nu_{k,n}}{\lambda_k^{max}}\Big) \geq \sum_{n \in N_k} log_2\Big(\frac{\nu_{k,n}}{\lambda_k^{max}}\Big)$$

Thus:

$$\sum_{n \in N_k} log_2(\frac{\nu_{k,n}}{\lambda_k}) \geq \sum_{n \in N_k} log_2\Big(\frac{\nu_{k,n}}{\lambda_k^{max}}\Big)$$

The proves that (by contradiction)

$$\lambda_k^{max} \leq \lambda_k$$

REFERENCES

[1] "Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update 2014–2019 White Paper." [Online]. Available: http://cisco.com/c/en/us/solutions/collateral/service-provider/visual-networking-index-vni/white_paper_c11-520862.html

[2] D. Feng, C. Jiang, G. Lim, J. Cimini, L.J., G. Feng, and G. Li, "A survey of energy-efficient wireless communications," *IEEE Communications Surveys Tutorials*, vol. 15, no. 1, pp. 167–178, 2013.

[3] G. Miao, N. Himayat, G. Li, and S. Talwar, "Distributed Interference-Aware Energy-Efficient Power Optimization," *IEEE Transactions on Wireless Communications*, vol. 10, no. 4, pp. 1323–1333, Apr. 2011.

[4] G. Bacci, E. V. Belmega, and L. Sanguinetti, "Distributed energy-efficient power and subcarrier allocation for OFDMA-based small cells," in *Communications Workshops (ICC), 2014 IEEE International Conference on*. IEEE, 2014, pp. 647–652. [Online]. Available: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=6881272

[5] D. Goodman and N. Mandayam, "Power control for wireless data," *IEEE Personal Communications*, vol. 7, no. 2, pp. 48–54, Apr. 2000.

[6] G. Bacci, E. V. Belmega, P. Mertikopoulos, and L. Sanguinetti, "Energy-Aware Competitive Power Allocation for Heterogeneous Networks Under QoS Constraints," *IEEE Transactions on Wireless Communications*, vol. 14, no. 9, pp. 4728–4742, Sep. 2015.

[7] Z. Zhou, M. Dong, K. Ota, J. Wu, and T. Sato, "Energy Efficiency and Spectral Efficiency Tradeoff in Device-to-Device (D2d) Communications," *IEEE Wireless Communications Letters*, vol. 3, no. 5, pp. 485–488, Oct. 2014.

[8] G. Bacci, E. V. Belmega, and L. Sanguinetti, "Distributed energy-efficient power optimization in cellular relay networks with minimum rate constraints," in *2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2014, pp. 7014–7018.

[9] S. Lasaulce, M. Debbah, and E. Altman, "Methodologies for analyzing equilibria in wireless games," *IEEE Signal Processing Magazine*, vol. 26, no. 5, pp. 41–52, Sep. 2009.